Robust Observer-Based Controller Design for Constrained Takagi-Sugeno Systems

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Abstract: This work concerns a descriptor Takagi-Sugeno (T-S) fuzzy system. The system is subject to disturbances, sensor faults, the presence of immeasurable premise variables, and input saturation which the latter are considered uncertainties of the system. First, the considered system is transformed into the polytopic representation, and then, a robust $\mathfrak{L}_2$ Proportional-Integral (PI) observer-based controller is designed. This observer-based controller, can estimate both the system states, and the faults, and stabilize closed-loop state trajectories. An integrated robust controller strategy is adopted by a novel structure of non-Parallel Distributed Compensation (non-PDC) control law. Furthermore, the proposed approach expresses the stability conditions of controller design are expressed in terms of Linear Matrix Inequalities (LMIs) constrained optimization problem. Using LMI TOOLBOX in MATLAB, we can easily design the type PI observer for efficient robust state/fault estimation and solve the $\mathfrak{L}_2$ observer-based controller design problem of discrete-time T-S fuzzy systems. Finally, an application to a rolling disc process is given to illustrate the effectiveness of the proposed method.

Keywords: Takagi-Sugeno (T-S) fuzzy system, Fault Estimation (FE), robust $\mathfrak{L}_2$PI Observer, unmeasurable premise variables, LMIs

Throughout the paper, the following notations are considered:

•$h_i(.)$,$v_{\ell}(.)$, $i \in \{1 \dots . r\}$, $\ell \in \{1 \dots . r_e\}$ *are the nonlinear scalar functions.*

• I , *denotes the identity matrix and* I_r, I_{r_e}. $\mathcal{N}_n$ *denotesthe set* $\{1,2,\cdots,r\}$,$\{1,2,\cdots,r_e\}$,$\mathfrak{R}^+$ *represents the set of positive real integer*,$\{1,\cdots,n\}$ *respectively*

•*The symbol* $*$ *indicates the transposed element in the symmetric positions*

• $\mathcal{H}(A)$ *denotes the Hermitian of the matrix A, i.e.* $\mathcal{H}(A) = A + A^T$.

• $\mathbb{Z} + (*)$*denotes* $\mathbb{Z} + \mathbb{Z}^T$.

• $\mathcal{X}^T > 0$ *means that* $\mathcal{X}$ *is a symmetric positive definite matrix.*

1. INTRODUCTION

Nowadays, the ordinary T-S fuzzy model has gained significant attention as a powerful tool in dynamic modeling. This model, extensively studied in[5] and [6], has been widely recognized for its ability to capture the global behavior of complex nonlinear systems by employing a set of local linear models interconnected through weighting functions. These weighting functions ensure a smooth transition between the contributions of each submodel, enabling an accurate representation of the system dynamics. One of the key advantages of the T-S fuzzy model is its compatibility with analysis and design tools developed for observer and controller synthesis in the linear case, as discussed in [7] and [8]. Notice that, the T-S explicit model [5] is a special case of the T-S descriptor model. In [9], [10], a fuzzy descriptor model is defined by extending the T-S fuzzy model [5]. Recall that, descriptor models also called singular models or implicit models, or differential-algebraic equations (DAEs) in the literature, have great theoretical and practical importance because they can describe the behavior of many chemical and physical processes see for instance [1], [2], [3], [4] for some real applications of descriptor models. The numerical simulation of such models typically involves combining an ordinary differential equation solver with an optimization algorithm, as stated in the literature. This approach allows for an effective analysis and design of these complex systems. Meanwhile, the field of observer design for dynamic systems has been the subject of extensive research for a considerable period. Observers play a crucial role in control, fault detection and diagnosis (FDD), and fault tolerant control (FTC) applications. Due to economic and security considerations, FDD and FTC problems have received significant attention in the study of chemical and physical processes. A range of approaches have been explored in the literature, including references [11], [12], [13], [14], [15], which provide comprehensive insights into these areas.

As a result, the theory of nonlinear control systems may be widely applied to evaluate and synthesize nonlinear systems stated as a multiple model method [3]. Therefore, there is an increasing interest in FE and FTC issues for a nonlinear system using the T-S fuzzy approach, and several significant findings have been reported in the literature [5-7], [16].

In the field of observer design for fault diagnosis, only a few works exist treating the problem of sensor or actuator faults management when system modeling is subject to uncertainties or time-varying parameters. In the literature, the term uncertainty is defined as the model parameters [7], [8], the model inputs [9], or the computer implementation [10].

In this paper, the presented work focuses on the observer design for discrete-time T-S fuzzy models with unknown input (UI) and disturbances, subject to states and input constraints.The FE and FTC have received great attention over the last several decades to increase system dependability and ensure system stability, and numerous results from these study domains have been described in the literature [9], [12]. It is difficult to use FE/FTC results directly to develop fault observers and FEs for nonlinear systems. Several least variance observers for such t-s systems have been proposed for exogenous disturbances. However, in this research, the disturbances are supposed to have finite energy. Hence, the use of $\mathfrak{L}_2$observer is particularly appropriate, as this type of observer seeks to reduce the transfer of energy between the disturbances and the estimation error [16-17].

The fundamental contribution of this study is the creation of an observer in charge of sensor fault estimate as well as system state concurrently, with less disturbances than the one presented at system dynamics. As a result of the fault estimation and estimated states, the suggested observer may be employed in a resilient framework to provide a saturated control signal with minimal disturbance corruption. This work investigates the observer design for a T-S system with unmeasurable premise variables using $\mathfrak{L}_2$ approach. Despite the presence of faults, the main goal is to address the FE problem for a class of constrained T-S fuzzy models subject to sensor and actuator faults [23]. This reconfigurable controller can also be designed to maintain stability, acceptable dynamic performance, and steady state of the overall system, as well as reduce conservatism compared to previous work, such as [6,16 and 21]. The outline of the paper is as follows: The structure of the considered class of T-S descriptor models in presence of sensor fault is presented in section 2. the key outcome of the proposed robust design$\mathfrak{L}_2$ PI observer permitting to estimate unmeasurable states and unknown faults of sensors is established in section 3. Finally, in section 4, Using a rolling disc descriptor model, we demonstrate the performance of the suggested observer design via simulation. [22-27]

II.TAKAGI-SUGENO DESCRIPTOR MODELS

In the following section, the controller is derived using the descriptor form. Sufficient LMI constraints are derived from Lyapunov's theory. Compared to [24], in the following section a constrained controller is proposed, for this we consider the following class of T-S fuzzy model, subject to input saturation, external disturbances, sensor fault

$$\begin{cases} \mathbb{E}x_{k+1} = \mathbb{A}x_k + \mathbb{B}sat(u_k) + B_d d_k \\ y_k = Cx_k + F_s f_{s,k} \end{cases} \quad (1.a)$$

Where $x_k \in \Re^{n_x}$,$u_k \in \Re^{n_u}$,$\{f_{s,k} = f_k\} \in \Re^{n_f}$,$y_k \in \Re^{n_y}$, $d_k \in \Re^{n_d}$ are the state, control input, sensor fault, output vector, and the exogenous disturbances respectively. The state-space matrices E, A_i, B_i, C, B_d are of the appropriate dimensions F_s and the matrix *C* involved in (1.a) is assumed to be a full row rank, *k* is a current samples, where $i \in I_r$ represent the *i-th* linear right hand-side submodelof T-S fuzzy model (1.a). Besides, we assume that $\mathbb{A} = A_{\hat{h}} + \Delta A$; $\mathbb{B} = B_{\hat{h}} + \Delta B$ .The uncertain matrices $\Delta A \in \Re^{n_x \times n_x}, \Delta B \in \Re^{n_x \times n_u}$, corresponding to the *i-th* subsystem contains the bounded uncertain terms, which can be rewritten as:$\Delta A = H_a \mathfrak{K}_{a,k} N_a$; $\Delta B = H_b \mathfrak{K}_{b,k} N_b$, with H_a, H_b, N_a, N_b, are known constant matrices, and $\mathfrak{K}_{a,k}, \mathfrak{K}_{b,k}$ are unknown matrices functions bounded, for all index $\varepsilon = a, b$ and $i \in I_r$, a one has $\mathfrak{K}_{\varepsilon,k}^T . \mathfrak{K}_{\varepsilon,k} \leq I$. And:

$$\Delta A = \sum_{i=1}^{r} (h_i(z_k) - h_i(\hat{z}_k) A_i,$$

$$\Delta B = \sum_{i=1}^{r} (h_i(z_k) - h_i(\hat{z}_k)) B_i,$$

System (1.a) can be represented by a polytopic form:

$$\begin{cases} Ex_{k+1} = \sum_{i=1}^{r} h_i(\hat{z}_k)(\mathbb{A}_i \\ x_k + \mathbb{B}_i sat(u_k)) \ + B_d d_k \quad (1.a) \\ y_k = Cx_k + F_s f_k \end{cases}$$

where the membership functions are denote$h_i(\hat{z}_k)$and vary within the convex sets £:

$$£ = \{h_i(\hat{z}_k) \in \Re^r;\ h_i(\hat{z}_k) = [h_1(\hat{z}_k), \dots \dots, h_r(\hat{z}_k)]^T\} \quad (2.a)$$

Note that $h_i(\hat{z}_k)$depend on the variable $\hat{z}_k$ verifying the convex sum property, rewritten here by convenience:
$\sum_{i=1}^{r} h_i(\hat{z}_k) = 1$ and

The augmented form is adopted. The T-S system (1.a) can be equivalently rewritten in the following compact augmented form:

$$\begin{cases} E^* x_{k+1}^* = \mathbb{A}^* x_k^* + \mathbb{B}^* sat(u_k) \\ \quad +B_d^* d_k \qquad (3.a) \\ y_k = C^* x_k^* + F_s f_k \end{cases}$$

where $x_k^* = [x_k \quad x_{k+1}]^T \in \Re^{n_{x^*}}$; $E^* = diag[I \quad 0]$;
$B_d^* = [0 \quad B_d]^T$; $\mathbb{B}^* = [0 \quad B_{\hbar} + \Delta B]^T$

$$\mathbb{A}^* = \begin{bmatrix} 0 & I \\ (A_{\hbar} + \Delta A) & -E \end{bmatrix}. \ (3.b)$$

III. THE CONTROL PROBLEM

A. Control law

In order to satisfy the desired constrained controller performance, the augmented controller design, is defined for system (3.a):

$$\begin{cases} u_k = F_{\hbar}^* H_{\hbar}^{-1^*} \hat{x}_k^* \\ \psi(u_k) = u_k - sat(u_k) \ (4.a) \\ \psi(0) = 0 \end{cases}$$

where:$F_{\hbar}^* = [F_{\hbar} \quad 0]$;$H_{\hbar}^* = diag[H_{\hbar} \quad 0]$. (4.b)

The gains $F_{\hbar}^*$and $H_{\hbar}^*$ are matrices controllers to be determined,$\hat{x}_k^*$ is the estimated augmented state variable of x_k^*. The architecture of the proposed constrained robust controller design is based on the scheme depicted in figure 1.

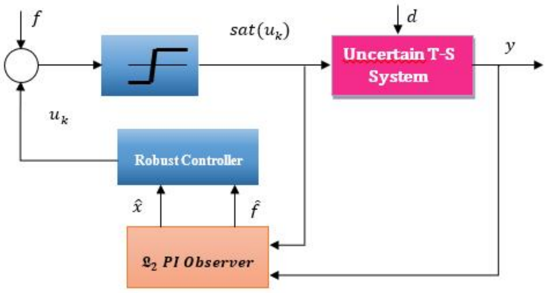

Fig 1. design of the proposed robust controller.

System (1.a) or (3.a) achieves observability conditions, as detailed in [21]. To develop the controller rules, a PI observer-based controller is synthesized to estimate both faults and system states (3.a)and has the augmented form:

$$\begin{cases} E^* \hat{x}_{k+1}^* = A_{\hbar}^* \hat{x}_k^* + B_{\hbar}^* u_k + L_{P\hbar}^*(y_k - \hat{y}_k) \\ \hat{y}_k = C^* \hat{x}_k^* + F_s \hat{f}_k \\ \hat{f}_{k+1} = \hat{f}_k - L_{I\hbar}(y_k - \hat{y}_k) \\ u_k = F_{\hbar}^* H_{\hbar}^{-1^*} \hat{x}_k^* \end{cases} \qquad (5.a)$$

Where $L_{P\hbar}^* = [0 \quad L_{P\hbar}]^T$(5.b)

$L_{P\hbar}^*$ is the augmented proportional gain for estimating the augmented variable state.

$L_{I\hbar}$is the integral gain for estimating the fault.

The estimation error between the system (4.a) and the observer (5.a) is given by:

$$e_{0,k}^* = x_k^* - \hat{x}_k^* (6)$$

The fault error is defined by:

$$e_{f,k} = f_k - \hat{f}_k \qquad (7)$$

In the following, substituting $u_k$of (4.a) into (3.a) we consider the augmented form as follows

$$\begin{cases} E^* x_{k+1}^* = \left(A_{\hbar}^* + B_{\hbar}^* F_{\hbar}^* H_{\hbar}^{-1^*}\right) x_k^* \\ -B_{\hbar}^* F_{\hbar}^* H_{\hbar}^{-1^*} e_{0,k}^* - B_{\hbar}^* \psi(u_k) + B_d^* d_k (8) \\ y_k = C^* x_k^* + F_s f_k \end{cases}$$

Now, using the augmented state vector is defined by: $X_k = [x_k^* \quad f_k]^T$(9), and system (8) becomes:

$$\begin{cases} \mathcal{E} X_{k+1} = \mathcal{A}_{\hbar\hbar} X_k - \mathcal{B}_{\hbar} \psi(u_k) + \mathcal{B}_d d_k \\ -B_{\hbar}^* F_{\hbar}^* H_{\hbar}^{-1^*} e_{0,k}^* \\ y_k = \mathrm{C} X_k (10.a) \\ u_k = \mathcal{F}_{\hbar} Y_{\hbar}^{-1} X_k \ - \ F_{\hbar}^* H_{\hbar}^{-1^*} e_{0,k}^* \end{cases}$$

With$\mathcal{E} = diag[E^* \quad 0]$; $\mathcal{B}_{\hbar} = [0 \quad B_{\hbar}^*]^T$;$\mathcal{B}_d = [B_d^* \quad 0]^T$;

$\mathcal{A}_{\hat{h}\hat{h}} = \begin{bmatrix} (A_{\hat{h}}^* + B_{\hat{h}}^* F_{\hat{h}}^* H_{\hat{h}}^{-1^*}) & 0 \\ 0 & 0 \end{bmatrix};$

$C = [C^* \quad F_s]; \mathcal{F}_{\hat{h}} = [F_{\hat{h}}^* \quad 0]; \Upsilon_{\hat{h}}\, diag[H_{\hat{h}}^* \quad 0]$ (10.b)

The dynamic error is defined by:

$E^* e_{0,k+1}^* = E^* x_{k+1}^* - E^* \hat{x}_{k+1}^* = (A_{\hat{h}}^* - L_{P\hat{h}}^* C^*) e_{0,k}^* - (L_{P\hat{h}}^* F_s) e_{f,k} - B_{\hat{h}}^* \psi_k + B_d^* d_k$ (11.a)

The dynamics of the fault estimation, with considering the fault constant $f_{k+1} \approx f_k$, the error is given by:

$e_{f,k+1} = f_{k+1} - \hat{f}_{k+1} =$
$f_k - \hat{f}_k + L_{P\hat{h}}^* C^* e_{0,k}^* + L_{P\hat{h}}^* F_s e_{f,k} = L_{I\hat{h}} C^* e_{0,k}^* + (L_{I\hat{h}} F_s + I_{e_f}) e_{f,k}$
(11.b)

Considering the dynamics (11.a) and (11.b), the augmented error dynamics is obtained as follows:

$$\begin{cases} E e_{k+1} = \mathfrak{A}_{\hat{h}\hat{h}} e_k - \mathcal{B}_{\hat{h}} \psi_k + \mathcal{B}_d d_k \\ u_k = \mathcal{F}_{\hat{h}} \Upsilon_{\hat{h}}^{-1} X_k - \mathcal{F}_{\hat{h}} \Upsilon_{\hat{h}}^{-1} e_k \end{cases} \quad (12.a)$$

With $E = diag[E^* \quad I_{e_f}]$;

$$\text{Ц}_{\hat{h}\hat{h}} = \begin{bmatrix} (A_{\hat{h}}^* - L_{P\hat{h}}^* C^*) & (-L_{P\hat{h}\hat{\vartheta}}^* F_s) \\ L_{I\hat{h}\hat{\vartheta}} C^* & (L_{I\hat{h}} F_s + I_{e_f}) \end{bmatrix} \quad (12.b)$$

Now, we consider the augmented system $\tilde{x}_k = [X_k \quad e_k]^T$, by combining the system (8) and the error dynamics (11), the closed-loop T-S system is obtained as follows:

$$\begin{cases} \tilde{E} \tilde{x}_{k+1} = \tilde{A}_{\hat{h}\hat{h}} \tilde{x}_k - \tilde{B}_{\hat{h}} \psi_k + \tilde{B}_d d_k \\ y_k = \mathcal{C} \tilde{x}_k \\ u_k = \tilde{\text{F}}_{\hat{h}} \tilde{\Upsilon}_{\hat{h}}^{-1} \tilde{x}_k \\ \psi(0) = 0 \end{cases} \quad (13.a)$$

$\tilde{\text{F}}_{\hat{h}} = [\mathcal{F}_{\hat{h}} \quad -\mathcal{F}_{\hat{h}}]$ and $\tilde{\Upsilon}_{\hat{h}} = diag[\Upsilon_{\hat{h}} \quad \Upsilon_{\hat{h}}]$; $\tilde{B}_{\hat{h}} = [\mathcal{B}_{\hat{h}} \quad \mathcal{B}_{\hat{h}}]^T$; $\tilde{B}_d = [\mathcal{B}_d \quad \mathcal{B}_d]^T$;

$$\mathcal{C} = [C \quad 0]; \tilde{A}_{\hat{h}\hat{h}} = \begin{bmatrix} \mathcal{A}_{\hat{h}\hat{h}} & -\mathfrak{Z}_{\hat{h}\hat{h}} \\ [0] & \text{Ц}_{\hat{h}\hat{h}} \end{bmatrix} . (13.b)$$

Moreover, to handle the non-linearity $sat(u_k)$, the dead-zone function $\psi(u_k)$ will be employed, where:

$$\psi(u_{k(l)}) = u_{k(l)} - sat(u_{k(l)}) \quad (14)$$

We use the generalized sector condition proposed by [20] to deal with the dead zone function. Also, the set $\tilde{\mathcal{D}}_u = \{\tilde{x}_k \in \Re^{n_{\tilde{x}}}: |u_{k(l)} - v_{k(l)}| \le u_{\max(l)}; l \in I_{n_l}\}$, with the auxiliary signal $v_{k(l)} = \tilde{\mathcal{W}}_{\hat{h}\hat{\vartheta}} \tilde{\Upsilon}_{\hat{h}\hat{\vartheta}}^{-1} \tilde{x}_k$ used as a degree of freedom in the design conditions,

.and the condition: $\psi(u_{k(l)})^T \tilde{S}_{\hat{h}\hat{\vartheta}(l)}{}^{-1} [\psi(u_{k(l)}) - v_{k(l)}] \le 0$ holds. Due to the saturating actuators, only initial conditions in a subset of $\mathcal{L}_v$ yield the trajectories of (3.a) to converge to the origin. Such a subset is denoted by $\mathcal{L}_v$, being called the Domainof Attraction (DoA). The determination of DoA is not an easy task even for small order systems since it can be non-convex, open, and in some cases, unbounded [15-18]. Therefore, an estimate of the DoA$\subseteq \tilde{\mathcal{D}}_u$ is computed, usually the largest possible. One way to construct the estimate DoA is to employ level sets taken from the Lyapunov function associated with the closed-loop system. To this end, a non quadratic Lyapunov function is considered: $V(\tilde{x}_k) = \tilde{x}_k^T \tilde{E}^T \tilde{P}_h^{-1} \tilde{E} \tilde{x}_k \le \rho$ (15), a level set associated with the Lyapunov function can be defined as in the following lemma:

Lemma 1. Suppose that $V(\tilde{x}_k)$ given in (15) is a Lyapunov function for system (13.a). Then, a possible level set is given by:

$\mathcal{L}_v = \bigcap_{\substack{\hat{z} \in \Omega_1 \\ \Omega_2}} \mathcal{E}(\tilde{E}^T \tilde{P}_h^{-1} \tilde{E}, \rho) = \bigcap_{i=1}^{r} \mathcal{E}(\tilde{E}^T \tilde{P}_i^{-1} \tilde{E}, \rho)$
(16)

For $\rho > 0$ and : $\mathcal{E}(\tilde{E}^T \tilde{P}_i^{-1} \tilde{E}, \rho) = \{\tilde{x}_k \in \Re^{n_{\tilde{x}}}: \tilde{x}_k^T \tilde{E}^T \tilde{P}_i^{-1} \tilde{E} \tilde{x}_k \le \rho\}$ (17)

For the proof, see of this lemma can be founded in [16].

Assumption 1. The state trajectories of T-S descriptor system (13.a) are contained within the following polyhedral set (validity domain), $\tilde{\mathcal{D}}_{\tilde{x}} \in \Re^{n_{\tilde{x}} \times n_{\tilde{x}}}$ defined as follows:

$$\tilde{\mathcal{D}}_{\tilde{x}} = \{\tilde{x}_k \in \Re^{n_{\tilde{x}}}: \tilde{\mathcal{N}}_m^T \tilde{x}_k \le 1, m \in I_{n_m}\} \quad (18.a)$$

Where the given matrix $\tilde{\mathcal{N}}_m \in \Re^{n_{\tilde{x}}}$, represents the state constraints of system (13.a), with:

$\tilde{\mathcal{N}}_m = [\mathcal{N}_m^* \quad 0_{n_u \times n_f} \quad 0_{n_u \times n_{e_0}} \quad 0_{n_u \times n_{e_f}}]$;

$\mathcal{N}_m^* = [\mathcal{N}_m \quad 0]$;(18.b)

B. LMI-Based design conditions of constrained descriptor system. This work is concerned with proposing a systematic method to design a controller such that the closed-loop system satisfies the following properties, presents a new LMI-based method to design an estimated constrained controller, besides a sufficient condition to solve the following control problem:

Property 1 *[local stability].* Given a scalar α', the initial condition $\tilde{x}(0)$ belong to a specific set in the state-space, which $(\tilde{E}^T \tilde{P}_h^{-1} \tilde{E}, \rho)$ is a region of asymptotic stability (RAS) for the saturated system. In the presence of disturbances, the controller guarantees that the trajectories of (13.a) are bounded, there exist a matrix $\tilde{P}_i > 0$ and a positive scalar $\rho > 0$ such that, for any $\tilde{x}(0) \in \mathcal{E}(\tilde{E}^T\tilde{P}_h^{-1}\tilde{E}, \rho)$ and $d_k \neq 0$, the trajectories of the saturated system remains inside the polyhedral set $\tilde{\mathcal{D}}_{\tilde{x}}$, and do not leave the ellipsoid $\mathcal{E}(\tilde{E}^T\tilde{P}_h^{-1}\tilde{E}, \rho)$, and converges exponentially to the equilibrium point with a decay rate less thanα', satisfying the following property:

Property 2 [$\mathfrak{L}_2$ *gain-performance*]. Given vector$\tilde{\mathcal{N}}_m$ defined in assumption 1, and a positive scalar δ depending in the type of disturbances involved in the dynamics of system (13.a), see [17] we distinguish two following control problems:

Control problem 1. When $d_k \neq 0$. There exist positive scalar ρ and γ such that $\forall\, \tilde{x}_k \in \frac{\mathcal{L}_v}{\{0\}}$, the corresponding closed-loop trajectory (13.a) remains inside the validity domain $\tilde{\mathcal{D}}_{\tilde{x}}$ defined in (18). Moreover the $\mathfrak{L}_2$-gain of the state vector $\tilde{x}_k$ is bounded as follows:

$$\|\tilde{x}\|_2^2 < \gamma^2\|d_k\|_2^2 + \rho, \forall k > 0 \quad (19)$$

Where the objective is to attenuate the effects of exogenous input d_k on the augmented state space by minimizing γ and ρ.

Control problem 2. Consider the T-S descriptor model design with a polytopic controller (13.a) such that: $\mathcal{L}_v \subseteq \tilde{\mathcal{D}}_{\tilde{x}} \cap \tilde{\mathcal{D}}_u$as large as possible, and is a contractively invariant set with respect to the closed-loop system.The problem is reformulated to design the observer/controller gains to guarantee asymptotic convergence to zero despite the mismatches.

The objective now is to compute the gains of observer based controller (5.a), to ensure the stability of the closed-loop system (13.a), guarantying the trajectories performance for all $d_k \neq 0$, sufficient conditions to achieve this objective are given through the following theorem:

Theorem 1. For a given the discrete-time T-S descriptor system (3.a) with a nonlinearities parameter $z_k \in$ £, under input saturation with the proposed observer based controller (5.a), whose validity domain is defined by $\tilde{\mathcal{D}}_{\tilde{x}}$, is locally exponentially stable if there exist a matrices$P_{i1} = P_{i1}^T > 0$, $P_{i2} = P_{i2}^T > 0$, $P_{i3} = P_{i3}^T > 0$, $P_{i4} = P_{i4}^T > 0$,$\{H_1, H_2, H_3, H_4\} \in \mathfrak{R}^{n_x \times n_x}$,$\{P_{i1}, P_{i2}\} \in \mathfrak{R}^{n_x \times n_x}$,$P_{i3}\mathfrak{R}^{n_f \times n_f}$,$P_{i4} \in \mathfrak{R}^{n_{ef} \times n_{ef}}$, $\{L_{P1}, L_{P2}L_{P3}L_{P4}\}\, \mathfrak{R}^{n_y \times n_x}\{L_{I1}, L_{I2}, L_{I3}, L_{I4}\} \in \mathfrak{R}^{n_f \times n_x}$for any diagonal gain matries$S_j \in \mathfrak{R}^{n_u \times n_u}$ a positive scalars $\tau_1, \tau_2, \tau_3, \tau_3, \tau_b, \bar{\gamma} = \sqrt{\gamma^2}, \rho, \eta$ where $(i, j) \in (I_r \times I_r)$,such that the following inequalities hold:

$$\begin{cases} \min \bar{\gamma}, \rho, \eta \\ \tilde{E}^{*T}P_i^{-1*}\tilde{E}^* > 0 \end{cases} \quad (20)$$

$$\rho + \gamma^2\delta < 1 \quad (21)$$

$$\begin{bmatrix} \Omega^{(1,1)} & * & * & * & * \\ 0 & P_{i2} & * & * & * \\ [0] & [0] & \Omega^{(1,1)} & * & * \\ 0 & 0 & 0 & P_{i4} & -u_{\max(l)}^2/\rho \\ \Gamma_{j(l)}^* & -W_{2j(l)} & \Gamma_{j(l)}^* & -W_{4j(l)} & \end{bmatrix} < 0 \quad (22)$$

$$\begin{bmatrix} \Omega^{(1,1)} & * & * & * & * \\ 0 & P_{i2} & * & * & * \\ [0] & [0] & \Omega^{(1,1)} & * & * \\ 0 & 0 & 0 & P_{i4} & * \\ \mathcal{N}_m^* H_j^* & 0 & [0] & 0 & -1/\rho \end{bmatrix} < 0$$

(23)

$$\begin{cases} \tau_{ii} < 0 \\ \frac{2}{r-1}\tau_{ii} + \tau_{ij} + \tau_{ji} < 0 \quad (i,j) \in (I_r \times I_r) \\ \qquad , i \neq j \end{cases}$$

(24)

Or τ_{ij} is defined in (25).

$$\tau_{ij} = \begin{bmatrix} \Omega + \mathrm{X}^T\mathfrak{Y}\mathrm{X} & * & * \\ \mathrm{Z}^T & -\mathfrak{Y}\mathrm{I} & * \\ \Theta_{12}^T & [0] & -\Theta_{22}I \end{bmatrix} < 0$$

(25.a)

$$\Omega + \mathrm{X}^T\mathfrak{Y}\mathrm{X} = \begin{bmatrix} \Omega^{(1,1)} & * & * & * & * \\ \tilde{\mathfrak{I}}_j^* & -I & * & * & * \\ \tilde{\mathcal{W}}_j & [0] & -2\tilde{S}_j & * & * \\ [0] & [0] & 0 & -\gamma^2 I & * \\ \tilde{A}_{ij}\tilde{\mathfrak{I}}_j^* & [0] & -\tilde{B}_i\tilde{S}_j & \tilde{B}_d & \Omega^{(5,5)} \end{bmatrix} < 0 \quad (25.b)$$

$$\mathrm{Z}^T = \begin{bmatrix} \tilde{N}_a^*\tilde{\mathfrak{I}}_j^* & [0] & [0] & [0] & [0] \\ [0] & [0] & \tilde{N}_b^*\tilde{S}_j^* & [0] & [0] \end{bmatrix} (25.c)$$

$$\mathfrak{Y} = \mathrm{diag}[\tilde{\tau}_a^* I \quad \tilde{\tau}_b^* I] (25.d)$$

$$\Theta_{12}^T = \begin{bmatrix} [L_j^* C^* & [0] & [0] & [0] & [0]] \\ [[0] & [0] & [0] & [0] & I] \end{bmatrix} \quad (25.e)$$

$$\Theta_{22} = \begin{bmatrix} \varepsilon\tilde{\mathfrak{I}}_j^{-1*} & * \\ [0] & \varepsilon^{-1}\tilde{\mathfrak{I}}_j^{-1*} \end{bmatrix} \quad (25.f)$$

$\tilde{\tau}_a^* = diag[\tau_1^* \quad \tau_1^* \quad \tau_3^*]$;$\tilde{\tau}_b^* = \tau_b$;

$\tau_1^* = diag[\tau_1 \quad \tau_2]$;

$\tau_3^* = diag[\tau_3 \quad \tau_4]$.
(25.g)

$\widetilde{\mathcal{W}}_j^* = [\mathrm{W}_{1j}^* \quad W_{2j} \quad \mathrm{W}_{1j}^* \quad W_{4j}]$.
(25.h)

$$\Omega^{(1,1)} = -\mathcal{H}(E^* H_j^*) + P_{i1}^*$$
$$\Gamma_{j(l)}^* = F_{j(l)}^* - \mathrm{W}_{j1(l)}^*$$
$$\Omega^{(5,5)} = -\tilde{P}_i + \tilde{\tau}_a^* \widetilde{H}_a^T \widetilde{H}_a^* + \tilde{\tau}_b^* \widetilde{H}_b^T \widetilde{H}_b^*$$

Proof. The demonstration is omitted for the sake of brevity.

IV. Application TO THE ROLLING DISC PROCESS

In this section, the approach of the proposed observer (5, a) is applied to a rolling disc process in order to ensure the simultaneous state and fault estimation. The T-S descriptor model given in [25] which is supposed to be affected by a sensor faults is then considered. It takes the form:

$$\begin{cases} E\dot{x} = \sum_{i=1}^{2} z_i(\mu)(A_i x + Bu) \\ \quad y = Cx + F_s f_s \end{cases} \tag{26}$$

where $x = (x_1, x_2, x_3; x_4)^T$ is the state vector with x_1 is the position of the center of the disc, x_2 is the translational velocity of the same point, x_3 is the angular velocity of the disc, x_4 is the contact force between the disc and the surface, u is the applied input force to the disc, y is the output measurement vector, and fs is the sensor fault.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ u_{max} & -b/m & 0 & 1/m \\ 0 & 1 & -r & 0 \\ u_{max} & -b/m \quad o & r^2/j + 1/m \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ u_{max} & -b/m & 0 & 1/m \\ 0 & 1 & -r & 0 \\ u_{max} & -b/m \quad o & r^2/j + 1/m \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, F_s = [1 \quad 0 \quad 0]^T$$

The weighting functions are given by:

$$\begin{cases} z_1 = \frac{\mu - \mu_{min}}{\mu_{max} - \mu_{min}} \\ z_2 = \frac{\mu_{max} - \mu}{\mu_{max} - \mu_{min}} \end{cases} (27)$$

As pointed out in [25], the expression of control variable u and values of physical parameters are from [26]. Under assumption 1, the expression of unknown fault signal fs is defined as in Figure3.

Therefore, to apply the proposed robust observer (5.a) for descriptor model (26), it suffices to rewrite the model (26) in its equivalent form (1.a) as mentioned above.

The resolution of LMIs of theorem 1, and the unknown gains of the PI observer based controller was done by choosing α = 1.5 and since the main purpose is to estimate the state variables and the faults, the value of matrix L_{Pj}, L_{Ij}, F_j and H_j are selected as:

$$H_1 = \begin{bmatrix} 2.8685 & 0 & 0.0001 & 0 \\ 0 & -2.2709 & -1.7331 & -0.1741 \\ 0.0001 & -1.7341 & -1{,}3557 & 0.5138 \\ 0 & -0.1748 & o.5131 & -13.0548 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} -3.1425 & 0 & 0.0002 & 0 \\ 0 & -2.9541 & 2.2639 & -0.0320 \\ -0.0002 & 2.2644 & -1.7680 & -0.6319 \\ 0 & -0.0302 & o.6331 & -13.2208 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} -3.1425 & 0.0042 & 0.0031 & 0.0004 \\ 0.0048 & -2.2575 & -1.7240 & -0.1752 \\ 0.0035 & -1.7238 & -1.3492 & 0.5220 \\ 0.0004 & -0.1765 & o.5208 & -13.2367 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} 3.1425 & 0.0061 & -0.0045 & 0.0001 \\ 0.0070 & -2.9542 & -2.2640 & -0.0320 \\ -0.0035 & 2.2644 & -1.7680 & -0.6319 \\ 0.0001 & -0.0302 & o.6331 & -13.2208 \end{bmatrix}$$

$F_1 = [0 \quad 0.0433 \quad 0.0319 \quad 0.0017]$;

$F_2 = F_3 = [0 \quad 0.0657 \quad -0.0496 \quad 0.0049]$

$F_4 = [0.0001 \quad 0.0426 \quad 0.0308 \quad 0.0016]$

$$LP_1 = LP_{2,3,4} = \begin{bmatrix} 0 & 0 & 0 \\ 0.0100 & 0 & 0 \\ -0.0006 & 0 & 0 \\ 0.3439 & 0.0002 & -0.0002 \end{bmatrix}$$

$LI_1 = LI_2 = [-0.9994 \quad 0 \quad 0]$;

$LI_3 = LI_4 = [-0.9996 \quad -0.0006 \quad 0.006]$

Which are used to construct the system (5.a) and are implemented in simulation. The scalars are: $\tau_{11} = 8.4268e + 04, \tau_{12} = -2.5834, \tau_{22} = -2.6112, \tau_{31} = 0.7719, \tau_{32} = -2.5846$, $\tau_b = -2.5\,851$, $\rho = 2.6021$, $\eta = 0.3612$, and $\bar{\gamma} = 2.6086$.

The input disturbance signal is defined as sinusoidal signal :

by$\omega = 0.1 * \sin(10t)$, depicted in Fig. 4. The discretization using Euler Lagrange is done with $T_e = 0.05[s]$. In this case, the measurements fault is

shown in Fig.1. Fig.3 shows states estimation, when we observe that the sensor fault is well attenuated. Fig.2 illustrates the output signal and its estimate, and converges towards zero. These simulation results show the applicability of the approach for estimating faults, states for T-S fuzzy systems that are stable despite the existence of faults and even with a large input saturation emerging at the beginning, as well as the disturbance and faults are well attenuated.

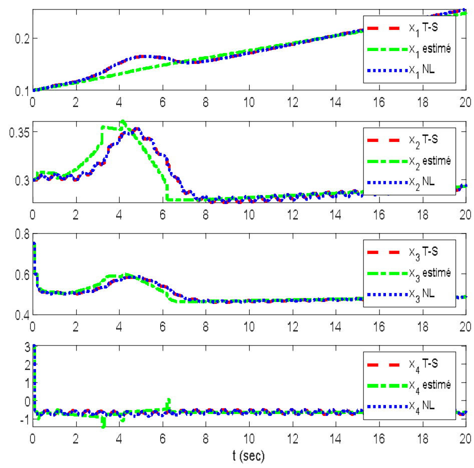

Fig1. states variables with their estimate.

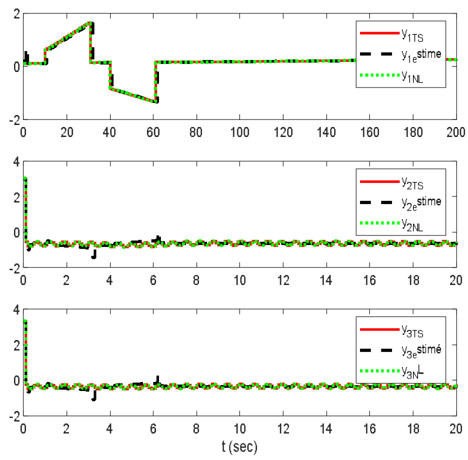

Fig. 2 the output signal NL, TS and its estimate.

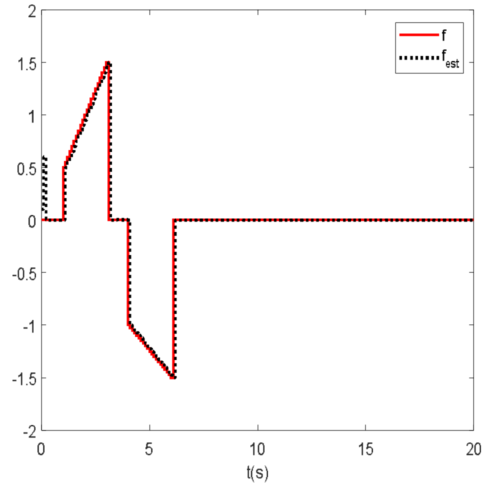

Fig3.faults with their estimates.

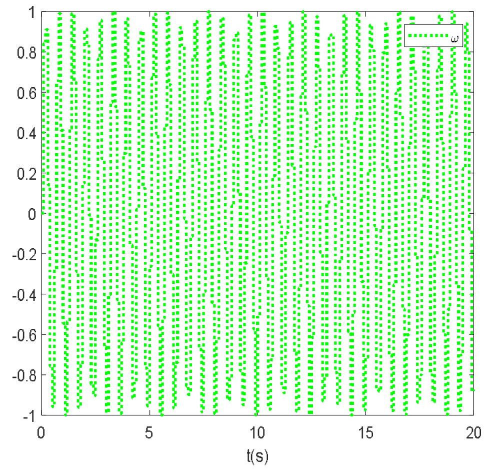

Fig4.the disturbance signal vector.

V. CONCLUSION

Based on a Robust $\mathfrak{L}_2$ PI observer design for a class of T-S descriptor models in presence of sensor faults and external disturbance, allowing the estimation of the unmeasurable states and the unknown faults simultaneously is proposed in this paper. The convergence of the state estimation error is studied by using the Lyapunov theory and the stability conditions are given in term of LMIs. The effectiveness of the proposed$\mathfrak{L}_2$ PIobserver is shown in simulation through a rolling disc process as an illustrative application.

References

[1] L. Dai. Singular Control Systems. Lecture Notes in Control and Information Sciences. Springer-Verlag. Berlin, 1989.

[2] A. Kumar and P. Daoutidis, Control of nonlinear differential algebraic equation systems. Chapman & Hall CRC, 1999.

[3] Chadli, M; « Stabilité et Commande de Systemes décrits par des Structures Multimodeles: Approche LMI » Thèse INPL-CRAN, Nancy, France, 2002

[4] G. R. Duan. Analysis and Design of Descriptor Linear Systems. Springer 2010.

[5] T. Takagi, M. Sugeno, Fuzzy identification of systems and its application to modeling and control. IEEE Trans. Syst., Man and Cyber, Vol.1115, pp. 116-132, 1985.

[6] T. Taniguchi, K. Tanaka, H. Ohtake and H. Wang, Model construction, rule reduction, and robust compensation for generalized form of Takagi-Sugeno fuzzy systems. IEEE Transactions on Fuzzy Systems, vol. 9, No.4, pp. 525-538, August 2001.

[7] K. Tanaka, and H. O. Wang, Fuzzy control systems design and analysis: A Linear Matrix Inequality Approach. John Wiley & Sons; 2001.

[8] Zs. Lendek, T. M. Guerra, R. Babuska and B. De Schutter. Stability analysis and nonlinear observer design using Takagi-Sugeno fuzzy models, Springer Berlin Heidelberg, 2011.

[9]S. J. Huang and G. H. Yang, "Fault Tolerant Controller Design for T–S Fuzzy Systems With Time–Varying Delay and Actuator Faults: A K-Step Fault–Estimation Approach," IEEE Trans. Fuzzy Syst., vol. 22, no. 6, 1526–1540, 2014.

[10] T. Taniguchi, K. Tanaka, and H. 0. Wang, Fuzzy Descriptor Systems and Nonlinear Model Following Control, IEEE Transactions on Fuzzy Systems, vol. 8, No.4, pp. 442-452, August 2000.

[11] R. Isermann, Fault-Diagnosis Systems An Introduction from Fault Detection to Fault Tolerance. Springer-Verlag Berlin Heidelberg 2006.

[12] Z. Lendek, T. M. Guerra, R. Babuka and B. De Schutter, Stability Analysis and Nonlinear Observer Design Using Takagi-Sugeno Fuzzy Models. Springer-Verlag Berlin Heidelberg 2010.

[13] M. Witczak, Fault Diagnosis and Fault-Tolerant Control Strategies for Non-Linear Systems. Analytical and Soft Computing Approaches. Springer International Publishing Switzerland 2014.

[14] M. Blanke, M. Kinnaert, J. Lunze, M. Staroswiecki, Diagnosis and Fault Tolerant Control. Springer-Verlag Berlin Heidelberg 2016.

[15] L. Li, Fault Detection and Fault-Tolerant Control for Nonlinear Systems. Springer Fachmedien Wiesbaden 2016.

[16]Aouaouda, S., &Chadli, M. (2015). Model Reference Tracking Control for Uncertain Takagi-Sugeno Systems subject to Sensor Faults. *IFAC-PapersOnLine*, *48*(21), 1250-1255.

[17]Tarbouriech, S., Garcia, G., da Silva Jr, J. M. G., &Queinnec, I. (2011). Stability and stabilization of linear systems with saturating actuators. Springer Science & Business Media.

[18]Escobet, T., Bregon, A., Pulido, B., & Puig, V. (2019). *Fault diagnosis of dynamic systems*. Springer International Publishing.

[19]Do, M. H., Koenig, D., &Theilliol, D. (2020). Robust H∞ proportional-integral observer-based controller for uncertain LPV system. *Journal of the Franklin Institute*, *357*(4), 2099-2130.

[20]S. Boyd, L. Ghaoui, and E. Feron, Linear Matrix Inequalities in Systems and Control Theory. Philadelphia, PA, USA: SIAM, 1994.

[21]Zhang, K., Jiang, B., Shi, P., &Cocquempot, V. (2018). *Observer-based fault estimation techniques*. Springer International Publishing.

[22]Righi, I., Aouaouda, S., Chadli, M., &Khelil, K. (2021). Robust controllers design for constrained nonlinear parameter varying descriptor systems. *International Journal of Robust and Nonlinear Control*, *31*(17), 8295-8328.

[23] D. Ichalal, B. Marx, J. Ragot, S. Mammar and D. Maquin. Sensor fault tolerant control of nonlinear Takagi-Sugeno systems: Application to vehicle lateral dynamics. Int. J. Robust NonlinearControl 2016; 26: 1376-1394. Published online 28 May 2015 in Wiley Online Library (wileyonlinelibrary.com).

[24]Guzman, J., López-Estrada, F. R., Estrada-Manzo, V., & Valencia-Palomo, G. (2021). Actuator fault estimation based on a proportional integral observer with nonquadratic Lyapunov functions. *International Journal of SystemsScience*, 1-14.

[25]M.Essabre, E. El Bouatmani, J. Soulami, A. El Assoudi, E. El Yaagoubi, Design of Fuzzy Observer for a Class of Takagi-Sugeno Descriptor Systems Subject to Unknown Inputs. The second International Conference on Intelligent Systems and Computer Vision, Fez, Morocco, April 17-19, 2017. 978-1-5090-4062-9/17/$ 31.00 2017 IEEE.